6.7 An infinitely long circular cylinder carries a uniform magnetization **M** parallel to its axis. Find the magnetic field (due to **M**) inside and outside the cylinder.

$$B = \mu_0 (H + M)$$

$$H = 0$$

$$B = \mu_0 M$$

$$B = \begin{cases} \mu_0 M & \text{inside the cylinder} \\ 0 & \text{out side the cylinder} \end{cases}$$

6.12 An infinitely long cylinder of radius R, carries a "frozen-in", magnetization parallel to its axis.

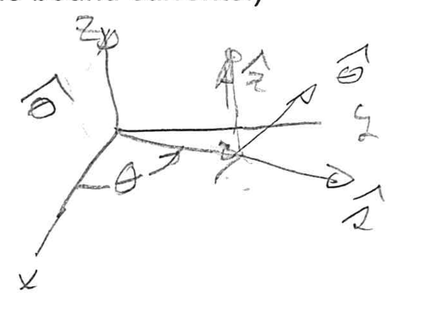
$$\mathbf{M} = k\, s\, \hat{z}$$

where k is a constant and s is the distance from the axis; there are no free electrons anywhere. Find the magnetic field inside and outside the cylinder by two different methods:

a. As in Sect. 6.2, locate all the bound currents, and calculate the field they produce.

b. Use Ampere's law (in the form of Eq. 6.20) to find **H**, and then get **B** from Eq.6.18. (Notice that the second method is much faster, and avoids any explicit reference to the bound currents.)

a) $J_b = \nabla \times M = \begin{vmatrix} \hat{z}/s & \hat{s}/s & \hat{\theta} \\ \frac{\partial}{\partial z} & \frac{\partial}{\partial s} & \frac{\partial}{\partial \theta} \\ M_z & 0 & 0 \end{vmatrix} = -\frac{\partial M_z}{\partial s}\hat{\theta}$



$\vec{M} = k s \vec{z} = M_z \vec{z}$ $\qquad \frac{\partial M_z}{\partial s} = k$

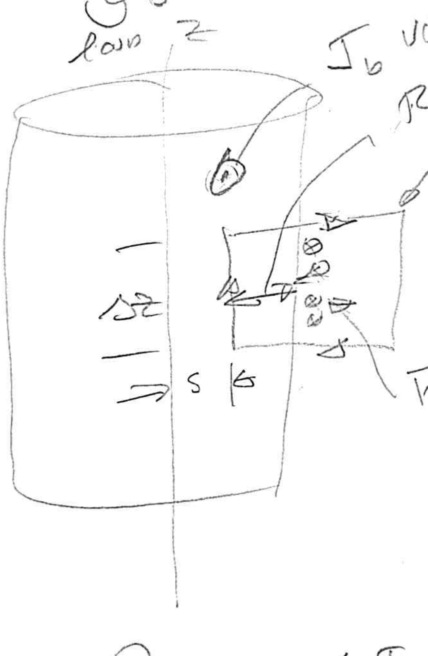
$\boxed{\vec{J}_b = -k\hat{\theta}}$ Amp/m^2

at the surface $\quad \vec{K} = \vec{M} \times \vec{n} = M(R)\,\hat{\theta} = kR\,\hat{\theta}$

$\boxed{\vec{K} = kR\vec{\theta} \text{ Amp/meter}}$

$\oint_{loop} B \cdot dl = \mu_0 I_{enc}$

J_b volume current out of paper $-\hat{\theta}$



horizontal legs cancel

ON Leg on RIGHT $B = 0 = M$

$B = \mu_0 (H + M)$

$H = 0 \quad B = \mu_0 M$

$\oint B \cdot dl = B_z\, \Delta z = \mu_0 I_{enc}$

$I_{enc} = I_{volume} + I_{surface}$

$I_{surface} = K \Delta z = kR\,\Delta z$

$\oint B \cdot dl = \mu_0 I_{enc}$

$I_{volume} = J\,\Delta z\,(R - s) = -k\,\Delta z\,(R-s)$

$B = \frac{\mu_0 I_{enc}}{\Delta z} = \frac{\mu_0}{\Delta z}\left[\cancel{kR\Delta z} - k\,\Delta z\,(\cancel{R} - s)\right] = \boxed{\mu_0 k s = B}$

$= 0 \quad s > R \qquad\qquad s < R$

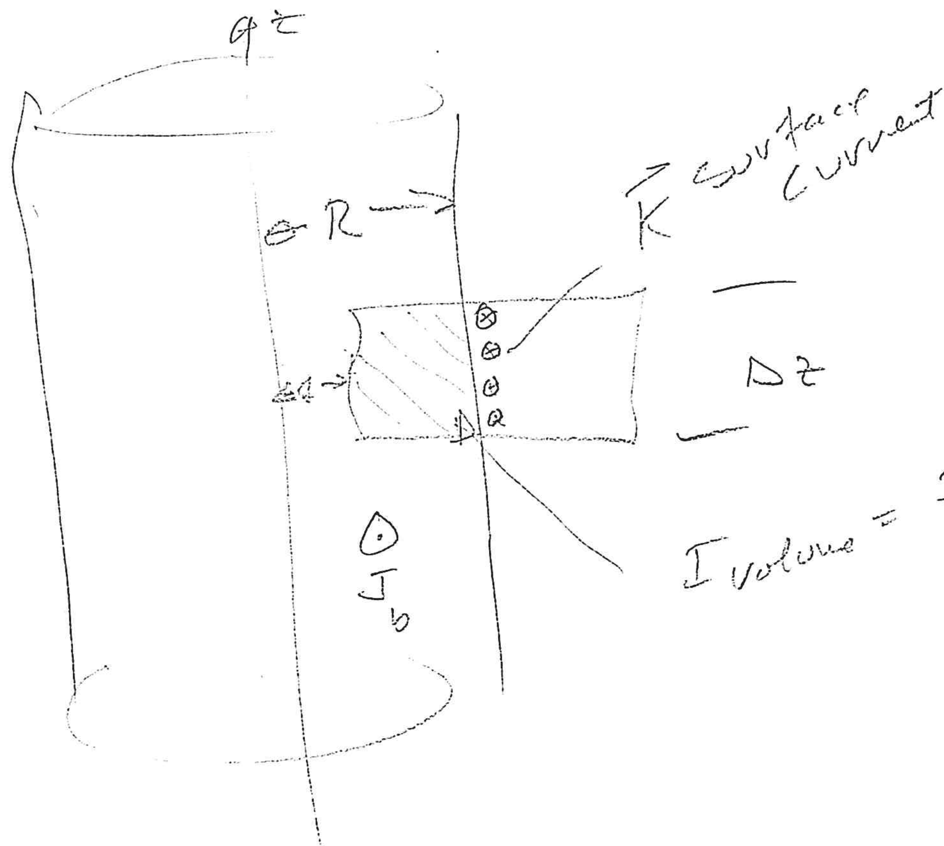
6.12 part b

$\oint H \cdot dl = I_{f_{enc}} = 0$ No free currents

$B = \mu_0 [H + M] = \mu_0 M = \mu_0 k s \hat{z}$ $s < R$

$$\vec{B} = \begin{cases} \mu_0 k s \hat{z} & s < R \\ 0 & s > R \end{cases}$$

In part a the total current enclosed is zero



$I_{volume} = J_b \Delta z (R - s)$

$J_b = -k \hat{\phi}$

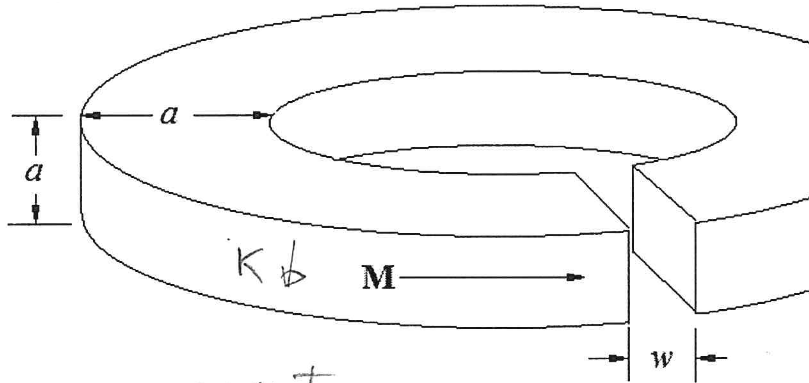
$I_{volume} = -k \Delta z (R$

The total Bound current is zero

$I_b = kR\Delta z - k \Delta z (R - s)$ $s = 0$ For total volume current

$I_{b_{TOTAL}} = 0$

6.10 An iron rod of length L and square cross section (*side* a), is given a uniform longitudinal magnetization **M**, and then bent around into a circle with a narrow gap (*width* w), as shown in Fig 6.14. Find the magnetic field at the center of the gap, assuming $w << a << L$). [*Hint:* treat it as the superposition of a complete torus plus a square loop with reverse current.]



A surface current

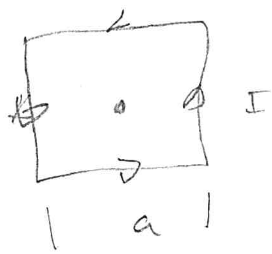
Figure 6.14

K_b flows as shown in Figure 6.14

$B = \mu_0(H+M) = \mu_0 M$ In side The Material

Account for the gap by placing a Loop of current opposing K so That the result is zero current flowing.

The magnetic field produced by a square Loop at the center



For one side

$$B = \frac{I\mu_0}{4\pi\, a/2}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right) = \frac{\mu_0 I\sqrt{2}}{4\pi a} \quad \frac{\mu_0 I\sqrt{2}}{2\pi a}$$

4 Sides Add

$$B = \frac{2\mu_0 I\, 2\sqrt{2}}{\pi a}$$

$m = I a^2 = \ldots$

$M = \frac{m}{Vol} = \frac{I a^2}{a^2 w}$

$I = Mw$ $B = \frac{2\mu_0 w M\, 2\sqrt{2}}{\pi a}$

$I = w M$

This Field subtracts from $B = \mu_0 M$

In the GAP $B_{Total} = \mu_0 M\left[1 - 2\sqrt{2}\,\frac{w}{a\pi}\right]$

6.17 A Current I flows down a long straight wire of radius a. If the wire is made of linear material (copper, or aluminum) with a susceptability χ_m, and the current is distributed uniformly, what is the magnetic field a distance s from the axis? Find the bound currents. What is the net bound current flowing down the wire?

Linear $B = \mu_0(1+\chi_m)H = \mu H$

$M = \chi_m H$

$J = \frac{I}{\pi a^2}$

$\oint H \cdot dl = I_{f_{enclosed}} = \frac{I r^2}{a^2}$

$I_{enc} = J \pi r^2 = \frac{r^2}{a^2} I$

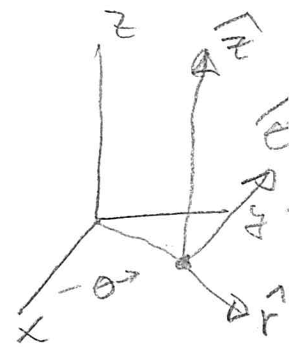
$H = \frac{1}{2\pi r} \frac{I r^2}{a^2} = \frac{I r}{2\pi a^2}$

$\vec{H} = \frac{I r}{2\pi a^2} \hat{\theta}$ $r < a$

$I_{enc} = I$ $r > a$

$\vec{H} = \frac{I}{2\pi r} \hat{\theta}$ $r > a$

$\vec{M} = \chi_m \vec{H} = \chi_m \frac{I r}{2\pi a^2} \hat{\theta}$



$$J_b = \nabla \times M = \begin{vmatrix} \hat{z}/r & \hat{r}/r & \hat{\theta} \\ 0 & \frac{\partial}{\partial r} & 0 \\ 0 & 0 & rM \end{vmatrix}$$

$$= \hat{z}\,\frac{1}{r}\frac{\partial}{\partial r}(rM) = \hat{z}\,\frac{1}{r}\left[M + r\frac{\partial M}{\partial r}\right]$$

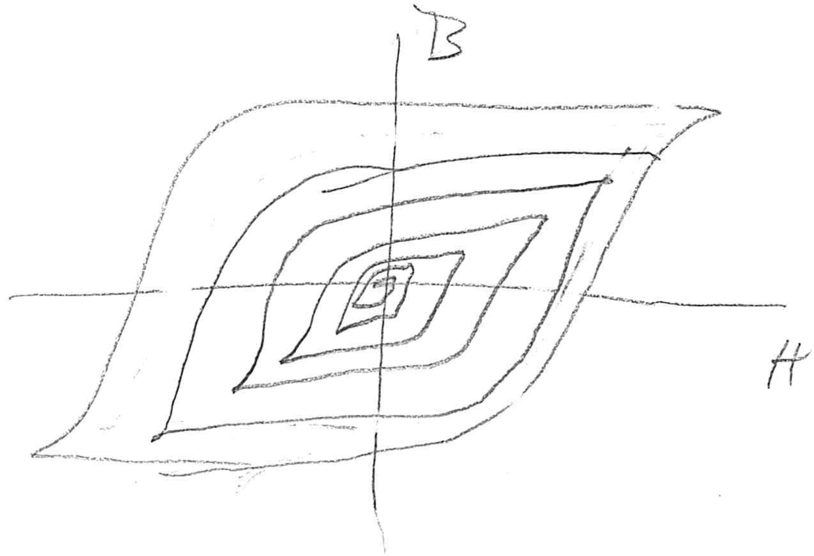
$$= \frac{\chi_m I}{2\pi a^2} + \frac{\partial M}{\partial r} = \frac{\chi_m I}{2\pi a^2} + \frac{\chi_m I}{2\pi a^2} = \frac{\chi_m I}{\pi a^2}$$ $r < a$

at the surface $\vec{K}_b = \vec{M} \times \vec{n} = -\chi_m \frac{I}{2\pi a} \hat{\theta}$

$$I_b = J_b \pi a^2 + K_b 2\pi a = \chi_m I - \chi_m I = 0$$

6.20 How would you go about demagnetizing a permanent magnet (such as the wrench we have been discussing, at point *c* in the hysteresis loop)? That is, how could you restore it to its original state, with $M = 0$ at $I = 0$?

$H = \frac{NI}{L}$



Excite the permanent magnet with an "Alternating current and gradually reduce the current as shown in drawing.