2.43 Find the capacitance per unit length of two coaxial metal cylindrical tubes, of radius *a* and *b* (Fig2.43).

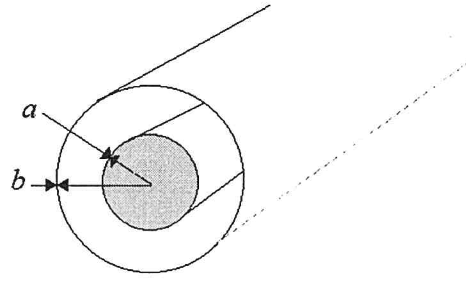


Figure 2.43

Assume $\lambda \frac{\text{coul}}{\text{m}}$ on the inner conductor

By Gauss' Law $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r} \quad a<r<b$

$$V = V(a) - V(b) = -\int_b^a \frac{\lambda\, dr}{2\pi\epsilon_0 r} = -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{a}{b}\right)$$

$$V = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right) \qquad \text{Since } -\ln\left(\frac{1}{x}\right) = \ln(x)$$

$$C = \frac{Q}{V}$$

$$\boxed{C/L = \frac{Q/L}{V} = \frac{\lambda}{V} = \frac{2\pi\epsilon_0}{\ln\frac{b}{a}}}$$

3.11 Two semi-infinite grounded conducting metal planes meet at right angles. In the region between them, there is a point charge, *q*, situated as shown in Figure 3.15. Set up an image configuration, and calculate the potential in this region. What charges do you need and where should they be located? What is the force on *q*? How much work did it take to bring *q* in from infinity? Suppose the planes meet at some angle other than 90°, would you still be able to solve the problem by the method of images? If not, for what particular angles *does* the method work?

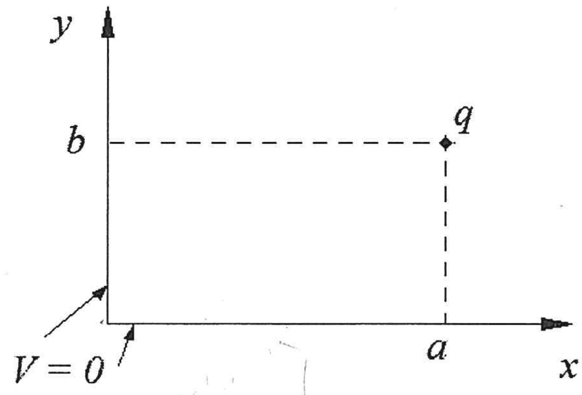
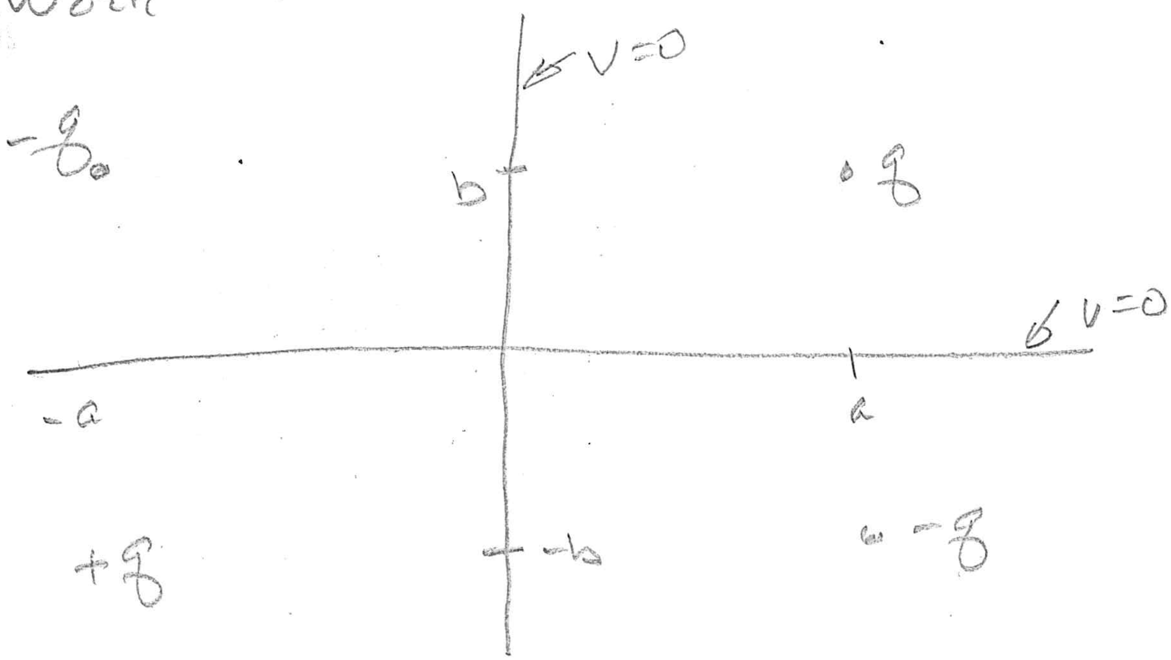


Figure 3.15

$\vec{F} = q\vec{E}$

Work $= \frac{1}{2} qV$



The voltage is zero if $x=0$ or if $y=0$
(on the axis)

The angle between the plates must be an integer divisor of 180°

i.e. 180, 90, 60, 45, etc

$\vec{F} = q\vec{E}$ $\qquad$ $\vec{F} = \frac{q^2}{4\pi\epsilon_0}\left[\hat{x}\left(\frac{-1}{(2a)^2}\right) + \hat{y}\left(\frac{-1}{(2b)^2}\right) + \frac{2a\hat{x} + 2b\hat{y}}{\left[(2a)^2+(2b)^2\right]^{3/2}}\right]$

Work $= qV/2$ $\qquad$ $W = \frac{q^2}{8\pi\epsilon_0}\left[\frac{-1}{2a} - \frac{1}{2b} + \frac{1}{\sqrt{a^2+b^2}}\right]$

4.6 A (perfect) dipole **p** is situated a distance *z* above an infinite grounded conducting plane. (Fig 4.7). The dipole makes an angle θ with the perpendicular to the plane. Find the torque on **p**. If the dipole is free to rotate, in what orentation will it come to rest?

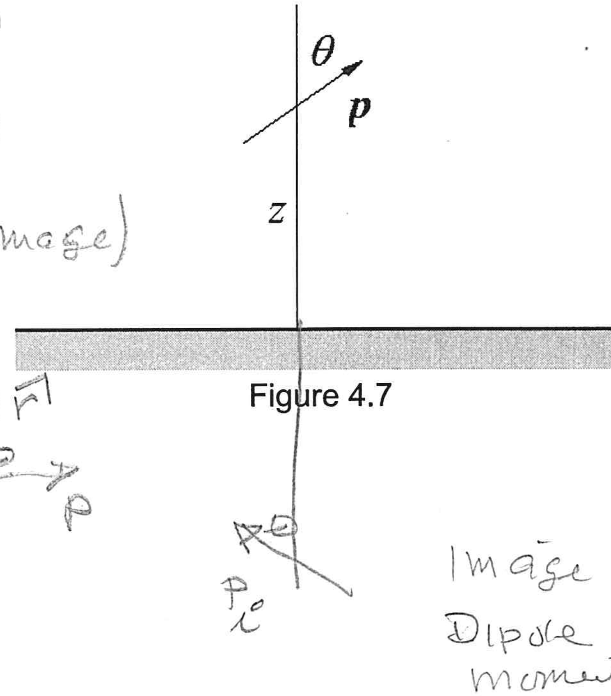
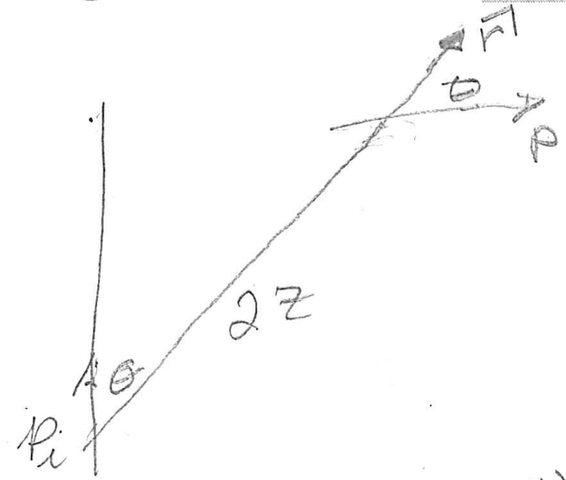


Figure 4.7

Redraw placing p_i (the image) at the origin



Voltage due to $\vec{p}_i$ = $V = \dfrac{\vec{p}_i \cdot \hat{r}}{4\pi\epsilon_0 r^2} = \dfrac{p\cos\theta}{4\pi\epsilon_0 r^2}$

In Spherical coordinates

$$\nabla V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\cancel{\frac{\partial V}{\partial \phi}}\hat{\phi}$$

0

$$\vec{E} = -\vec{\nabla}V = \frac{p}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

$r = 2z$

$\overrightarrow{\text{Torque}} = \vec{p} \times \vec{E}$

$\vec{p} = p\cos\theta\,\hat{r} + p\sin\theta\,\hat{\theta}$

$$\overrightarrow{\text{Torque}} = \vec{p} \times \vec{E} = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ \cos\theta & \sin\theta & 0 \\ 2\cos\theta & \sin\theta & 0 \end{vmatrix} \times \frac{p^2}{4\pi\epsilon_0 (2z)^3}$$

$$\boxed{\overrightarrow{\text{Torque}} = \frac{-p^2 \cos\theta \sin\theta}{4\pi\epsilon_0 (2z)^3}\,\hat{\phi}}$$

4.18 The space between the plates of a parallel-plate capacitor (Fig 4.24) is filled with two slabs of linear dielectric material. Each slab has a thickness *a*, so that the total distance between the plates is *2a*. Slab 1 has a dielectric constant of 2, and slab 2 has a dielaecric constant of 1.5. The free charge density on the top plate is σ and on the bottom plate is $-\sigma$

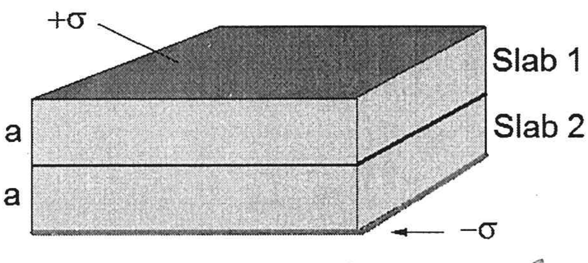
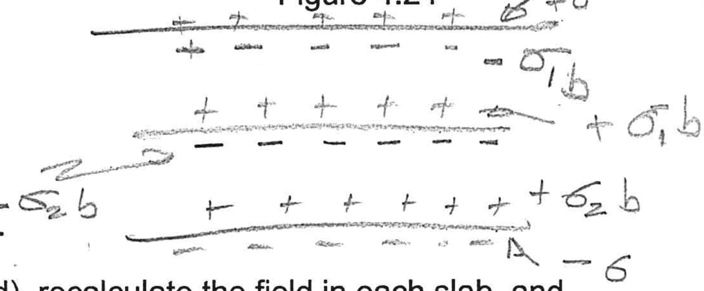


Figure 4.24



a. Find the electric displacement **D** in each slab.

b. Find the electric field in each slab.

c. Find the potential difference between the plates.

d. Find the location and amount of all bound charge.

e. Now that you know all the charge (free and bound), recalculate the field in each slab, and confirm your answer to (b).

a) $\oint \vec{D} \cdot d\vec{s} = Q_{enc} = \frac{\sigma}{A} = DA \quad D = \sigma$. Both SLABS

b) $E_1 = \frac{D}{\epsilon_1} = \frac{\sigma}{2\epsilon_0} \qquad E_2 = \frac{D}{\epsilon_2} = \frac{\sigma}{1.5\epsilon_0} = \frac{2}{3}\frac{\sigma}{\epsilon_0}$

c) $V = E_1 a + E_2 a = \frac{a\sigma}{\epsilon_0}\left[\frac{1}{2} + \frac{1}{1.5}\right] =$

$= \frac{a\sigma}{\epsilon_0}\left[\frac{1}{2} + \frac{2}{3}\right] = \frac{a\sigma}{\epsilon_0} * \frac{7}{6}$

d) $\sigma_b = P_N =$ Normal component of P

$D = \epsilon_r \epsilon_0 E = \epsilon_0 E + P$

$P = \epsilon_0(\epsilon_r - 1)E$

$E = \frac{D}{\epsilon_r \epsilon_0}$

$P = \frac{\epsilon_r - 1}{\epsilon_r} D = \left(\frac{\epsilon_r - 1}{\epsilon_r}\right)\sigma$

$P_1 = \frac{\sigma}{2} = \sigma_{b1}$

$P_2 = \frac{\sigma(0.5)}{1.5} = \frac{\sigma}{3} = \sigma_{b2}$

e) $E_1 = \frac{\sigma - \sigma_{b1}}{\epsilon_0} = \frac{\sigma}{2\epsilon_0} \qquad E_2 = \frac{\sigma - \sigma_{2b}}{\epsilon_0} = \frac{2}{3}\frac{\sigma}{\epsilon_0}$