2.14 Find the electric field inside a sphere that carries a charge density proportional to the distance from the origin, $\rho = k\,r$, for some constant, k. [Hint this charge density is not uniform, and you must integrate to get the enclosed charge.]

$$\oint \vec{E} \cdot d\vec{a} = E_r 4\pi r^2 = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{enc} = \int \rho \, d\tau = \int_0^r k r \, 4\pi r^2 \, dr$$

$$Q_{enc} = 4\pi k \int_0^r r^3 \, dr = 4\pi k \frac{r^4}{4} = \pi k r^4$$

$$\oint \vec{E} \cdot d\vec{a} = E_r 4\cancel{\pi} \cancel{r^2} = \frac{\cancel{\pi} k r^{\cancel{4}\,2}}{\epsilon_0}$$

$$\boxed{E_r = \frac{k r^2}{4 \epsilon_0}}$$

$$\boxed{\vec{E} = \frac{k r^2}{4 \epsilon_0} \hat{r}}$$

2.15 A thick spherical shell carries a charge density

$$\rho = \frac{k}{r^2} \qquad (a \le r \le b)$$

(Fig. 2.25) Find the electric field in three regions: (i) $r < a$, (ii) $a < r < b$, (iii) $r > b$. Plot |E| as a function of r, for the case $b = 2a$.

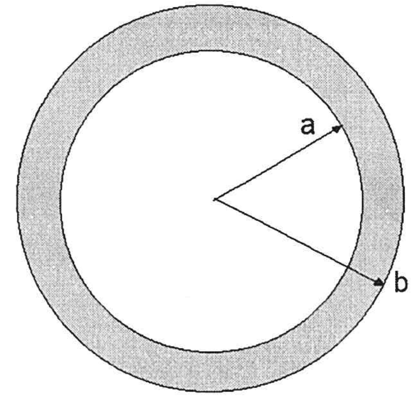


Figure 2.25

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{a} = E_r 4\pi r^2$$

(i) $r < a$ $Q_{enc} = 0 \therefore \vec{E} = 0$

(ii) $a < r < b$ $Q_{enc} = \int_a^r \rho \, d\tau$ $d\tau = 4\pi r^2 dr$

$$Q_{enc} = \int_a^r \frac{k}{\cancel{r^2}} 4\pi \cancel{r^2} dr$$

$$= 4\pi k [r - a]$$

$$\vec{E} = \frac{4\pi k [r-a]}{4\pi \epsilon_0 r^2} \hat{r} = \boxed{\frac{k[r-a]}{\epsilon_0 r^2} \hat{r}}$$

(iii) $r > b$ $Q_{enc} = 4\pi k [b - a]$

$$\boxed{\vec{E} = \frac{k[b-a]}{\epsilon_0 r^2} \hat{r}}$$

2.16 A long coaxial cable (Fig 2.26) carries a uniform *volume* charge density ρ on the inner cylinder (radius a), and a uniform *surface* charge on the outer cylindrical shell (radius b). This surface charge is negative and is of just the right magnitude that the cable as a whole is electrically neutral. Find the electric field in each of the three regions: (i) inside the inner cylinder ($s < a$), (ii) between the cylinders ($a < s < b$), (iii) outside the cable ($s > b$). *Plot |E| as a function of s.*

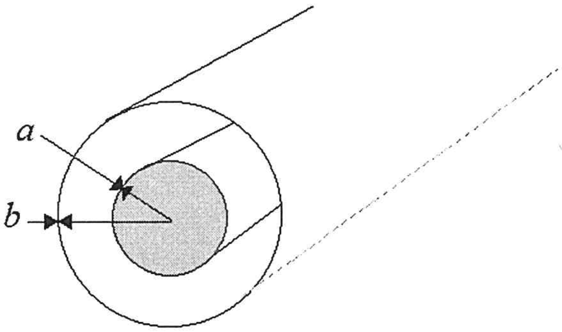
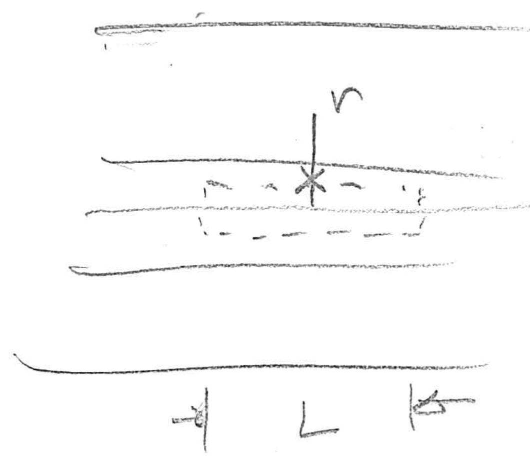


Figure 2.26

Cylindrical Symmetry

$$\oint \vec{E}\cdot d\vec{a} = E_r L 2\pi r$$



r < a

$$Q_{enc} = \int_0^r \rho \, d\tau$$

$d\tau = L 2\pi r\, dr \rightarrow$ Shell

$\rho =$ constant

$$Q_{enc} = \int_0^r \rho L 2\pi r\, dr = \rho L 2\pi \int_0^r r\, dr = \rho L 2\pi \frac{r^2}{2}$$

$$\oint \vec{E}\cdot d\vec{a} = E_r \cancel{L 2\pi r} = \frac{\cancel{\rho L 2\pi} r^2}{2\epsilon_0}$$

(i) $\boxed{\vec{E} = \frac{\rho r}{2\epsilon_0}\hat{r}}$

(ii) $a < r < b$ $\quad Q_{enc} = \rho L \pi a^2$

$$\vec{E} = \frac{Q_{enc}}{2\pi r L \epsilon_0}\hat{r} = \frac{\rho \cancel{L\pi} a^2}{2\cancel{\pi} r \cancel{L}\epsilon_0}\hat{r} = \boxed{\frac{\rho a^2}{2\epsilon_0 r}\hat{r}}$$

(iii) $Q_{enc} = 0 \quad \vec{E} = 0$

2.17 An infinite plane slab, of thickness, *2d*, carries a uniform charge density ρ (Fig 2.27). Find the electric field, as a function of *y*, calling *E* positive when it points in the +*y* direction and negative when it points in the -*y* direction.

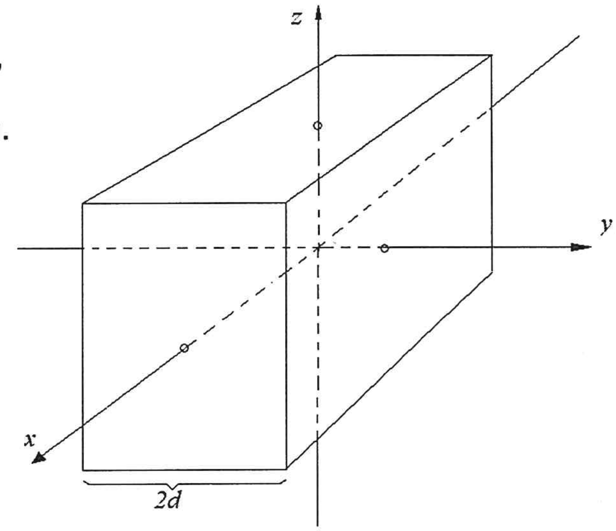


Figure 2.27

Due to Symmetry

$E = 0$ at $y = 0$

$\vec{E} = E_y \hat{y}$

$E_x = E_z = 0$

$\oint E \cdot da = A E_y = \frac{Q_{enc}}{\epsilon_0}$

$y < d \quad Q_{enc} = \int_0^y \rho \, d\tau'$

$d\tau = A\,dy$

$Q_{enc} = \int_0^y \rho A \, dy = \rho A y$ Area A

$\vec{E} = \hat{y} \frac{\rho A y}{A \epsilon_0} = \frac{\rho}{\epsilon_0} y \hat{y} \quad |y| < d$

$y > d \quad Q_{enc} = \rho A d$

$\vec{E} = \frac{\rho A d}{\epsilon_0 A} \hat{y} = \frac{\rho d}{\epsilon_0} \hat{y}$