7.12 A long solenoid, of radius *a*, is driven by an alternate current, so that the field inside is sinusoidal, ***B*** *(t)* = B_o *cos(ωt)* in the z direction. A circular loop of wire, of radius *a/2* and resistance *R*, is placed inside the solenoid, and coaxial with it. Find the current induced in the loop, as a function of time.

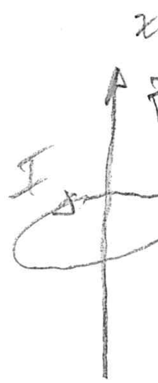
$$\Phi = B(t)\,A = B(t)\,\pi\left(\frac{a}{2}\right)^2$$

$$\Phi = \frac{\pi a^2}{4} B_o \cos(\omega t)$$

$$\frac{d\Phi}{dt} = -\frac{\pi a^2}{4} B_o \omega \sin(\omega t)$$

$$V = -\frac{d\Phi}{dt} = \frac{\pi a^2}{4} B_o \omega \sin(\omega t)$$

$$\boxed{I = \frac{V}{R} = \frac{\pi a^2 B_o \omega}{4R} \sin(\omega t)}$$



$$\vec{B} = \hat{z}\, B_o \cos \omega t$$

$$I = \hat{\phi}\, \frac{\pi a^2}{4R} B_o \omega \sin(\omega t)$$

For $0 < \omega t < \frac{\pi}{2}$

B is decreasing

∴ Φ is decreasing

By Lenz's Law Current flows counter clockwize about the z axis to produce a compensating B field in the z direction

7.17 A long solenoid of radius a, carring n turns per unit length, is looped by a wire with resistance R, as shown in Fig 7.27

a. If the current in the solenoid is increasing at a constant rate ($dI/dt = k$), what ~~what~~ current flows in the loop, and which way (left of right) does it pass through the resistor?

b. If the current I in the solenoid is constant but the solenoid is pulled out of the loop, turned around, and reinserted, what total charge passes through the tesistor?

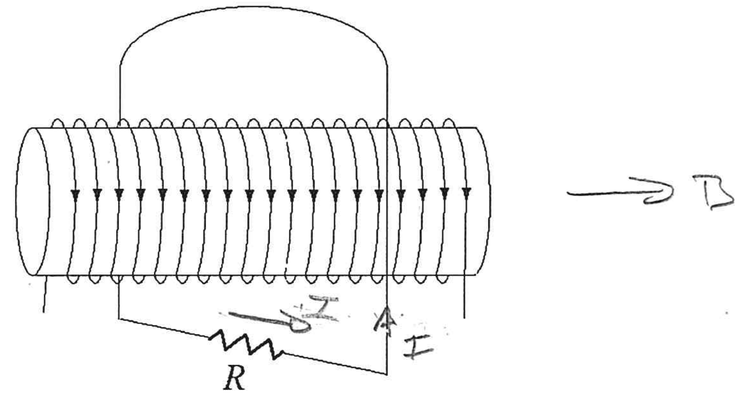


Figure 7.27

$B = \mu n I$

$\Phi = BA = B\pi a^2 = n\pi \mu_0 a^2 I$

$V = \frac{d\Phi}{dt} = \mu_0 n \pi a^2 \frac{dI}{dt} = \mu_0 n \pi a^2 k$

$I = V/R = \frac{\mu_0 n \pi a^2 k}{R}$

Current flows from the left to the right to produce a B field that compensates for the increasing B field produced by the solenoid

a)

b) $\Delta\Phi = 2\Phi = 2n\pi\mu_0 a^2 I$

$I = \frac{dQ}{dt} = \frac{V}{R} = \frac{1}{R}\frac{d\Phi}{dt}$

$dQ = \frac{d\Phi}{R}$ $\quad \Delta Q = \frac{2n\pi\mu_0 a^2 I}{R} = \frac{\Delta\Phi}{R}$

7.24 Find the self-inductance per unit length of a long solinoid, of radius R, carring n turns per unit length.

$$B = \mu_0 n I$$

$$\Phi = \mu_0 n I \pi R^2$$

$$n\Phi = \mu_0 n^2 I \pi R^2 = LI$$

$$\boxed{L = \mu_0 n^2 \pi R^2}$$

7.34 Suppose the circuit in Fig 7.40 has been connected for a long time when suddenly, at time $t = 0$, switch S is thrown, bypassing the battery.

a. What is the current at any subsequent time t?

b. What is the total energy delivered to the resistor?

c. Show that this is equal to the energy originally stored in the inductor.

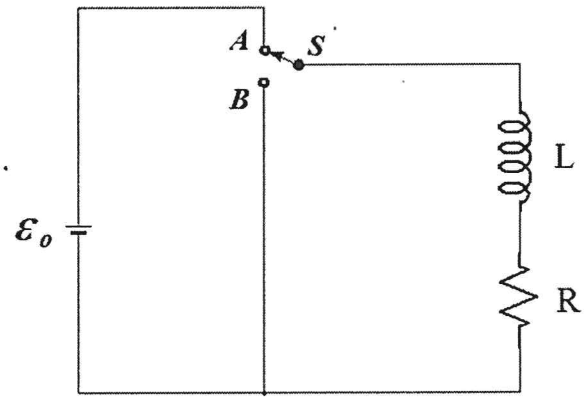
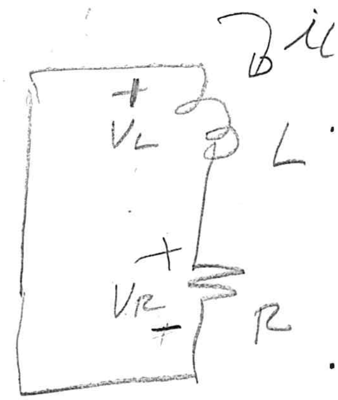


Figure 7.40



$V_R + V_L = 0$

$iR + L\frac{di}{dt} = 0$

$\frac{L}{R}\frac{di}{dt} + i = 0$

Let $i = Ae^{at}$

$\frac{di}{dt} = ai$

$\frac{L}{R}ai + i = 0$

$a = -\frac{R}{L}$

$i = Ae^{-\frac{R}{L}t} = Ae^{-t/\tau}$ $\tau = \frac{L}{R}$

at $t = 0$ $i = \frac{\mathcal{E}_0}{R} = A$

$\boxed{i = \frac{\mathcal{E}_0}{R}e^{-t/\tau}}$

1.34 CONTINUED

$$W_R = \int_0^{\infty} i v_R \, dt = \int_0^{\infty} i^2 R \, dt$$

$$i = I_0 e^{-t/\tau} \qquad I_0 = \frac{\mathcal{E}_0}{R}$$

$$W_R = R I_0^2 \int_0^{\infty} e^{-2t/\tau} dt$$

$$= R I_0^2 \left[\frac{e^{-2t/\tau}}{-\frac{2}{\tau}} \right. = \frac{R I_0^2 \tau}{2}$$

$$\tau = \frac{L}{R}$$

$$W_R = I_0^2 \frac{R}{2} \frac{L}{R}$$

$$\boxed{W_R = \frac{1}{2} L I_0^2 = W_L}$$