PHYS 350 Intermediate Electromagnetics
Final

April 29, 2017

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1. Write Maxwell's equations in;
    a. Differential form,
    b. Integral form,
    c. Words.

a) $\nabla \cdot \vec{E} = \rho/\epsilon_0$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\nabla \cdot \vec{B} = 0$

$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

b) $\oint_S \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} \int_V \rho \, d\tau$

$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$

$\oint \vec{B} \cdot d\vec{a} = 0$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{a} + \mu_0 \epsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{a}$

c) (i) Gauss' Law: The total electric flux out of a closed surface is equal to the charge enclosed divided by ϵ_0

(ii) Faraday's Law: The line integral of the electric field around a closed loop is equal to the negative change in the magnetic flux through the loop with time.

(iii) The total magnetic flux out of a closed volume is zero.

(iv) Amperes' Law: The line integral of the magnetic field around a closed loop is equal to the current flowing through the loop.

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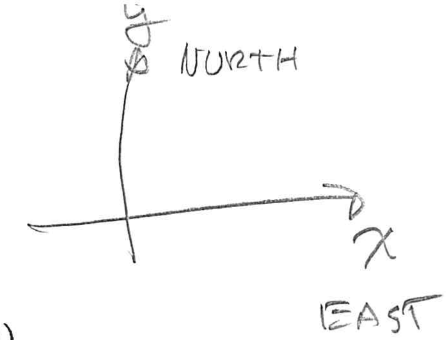

2. The height of a certain hill (in feet) is given by

$$h(x, y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$$

where *y* is the distance (in miles) north, *x* is the distance east of South Hadley.

a. Where is the top of the hill located?

b. How high is the hill?

c. How steep is the hill (in feet per mile) at a point 1 mile north and one mile east of South Hadley? In what direction is the slope steepest at that location?

a) Top is where $\vec{\nabla} h = 0$

at the Top $\vec{\nabla} h = \hat{x}\frac{\partial h}{\partial x} + \hat{y}\frac{\partial h}{\partial y} = 10\hat{x}(2y - 6x - 18) + 10\hat{y}(2x - 8y + 28)$

$\frac{\partial h}{\partial x} = 0 \quad \frac{\partial h}{\partial y} = 0$

$2y - 6x - 18 = 0$ EQ A

$-8y + 2x + 28 = 0$ EQ B

Multiply EQ A By 4 And Add To EQ B

$8y - 24x - 72 = 0$

$-22x - 44 = 0$

$x = -2$

Plug into EQ A $2y + 12 - 18 = 0$

$2y = 6$

$y = 3$

at the Top

$x = -2$

$y = 3$

Top @ 2 Miles West and 3 miles North of North Hadley

b) Height $= h(-2, 3) = 10(-12 - 12 - 36 + 36 + 84 + 12$

-12

72

Height = 720 feet

90
45
5

c) Slope $= \vec{\nabla} h = 10\hat{x}(2 - 6 - 18) + 10\hat{y}(2 - 8 + 28)$

$= -220\hat{x} + 220\hat{y} = 220\sqrt{2}\left(\frac{-\hat{x} + \hat{y}}{\sqrt{2}}\right)$

$= 311 \angle 135°$

The hill is 311 ft/mi Steep at an angle 45° West of North

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3. Suppose the circuit in Figure 3 has been connected to point ***A*** for a long time when suddenly, at time $t = 0$, switch S is thrown to position ***B***.

   a. What is the current at any subsequent time t? ____________________

   b. How much energy is disipated in the resistor during this time? ________________

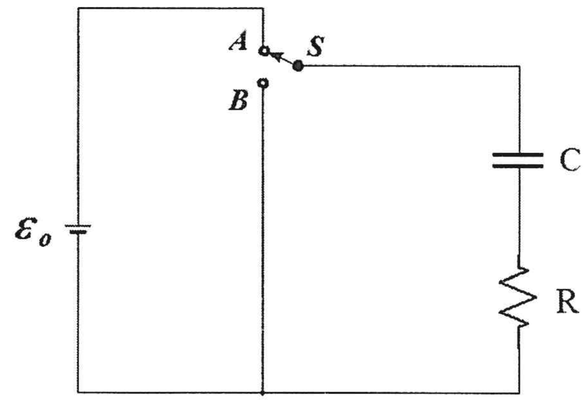
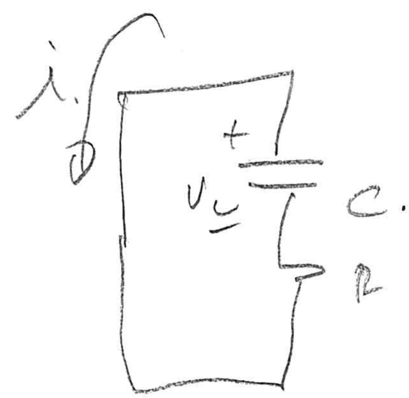


Figure 3

at $t = 0^+$ $V_c = \mathcal{E}_0$

@ $t = 0$ $i = \frac{\mathcal{E}_0}{R}$

a) $i = \frac{\mathcal{E}_0}{R} e^{-t/RC}$



b) the energy disipated in the resistor is equal to the energy stored in the capacitor. $W = \frac{1}{2} C V^2 = \frac{1}{2} C \mathcal{E}_0^2$

at $t = 0$

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4. The conducting bar in Figure 4 slides on conducting rails at a constant velocity. If **B** = 0.5 Tesla out of the paper, and L = 0.2 m,

    a. What force is required to maintain the constant velocity? $.01v/R$ Newton

    b. What is the power dissipated in the resistor? $.01v^2/R$ $(Blv)^2/R$

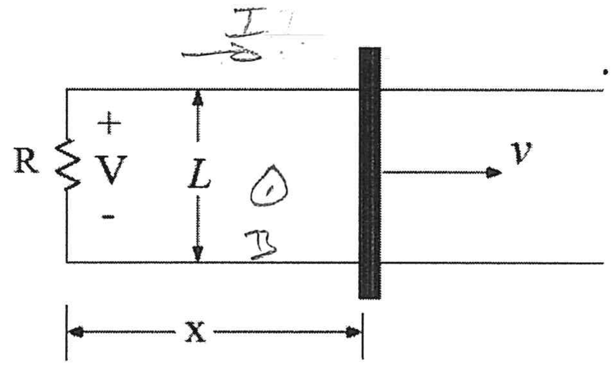

Figure 4

$V = Blv$

$I = \frac{V}{R} = \frac{Blv}{R}$

$F = IlB = \frac{(Bl)^2 v}{R}$

$F = qv \times B$

$dF = I\,dl \times B$

$F = IlB$

Voltage $= \frac{\partial \Phi}{\partial t}$

$V = Blv$

$\phi = BA = Blx$ out of paper

By Lenz's Law current will flow to cancel the increase in flux

That is a clockwise current.

Down through the BAR

B I $I \times B$ F

The Force produced by the current in the B Field is to the left

a) $F = \frac{(Bl)^2 v}{R} = \frac{(0.1)^2 v}{R} = \frac{.01 v}{R}$ Newtons

b) Power $= \frac{V^2}{R} = \frac{(Blv)^2}{R} = \frac{.01 v^2}{R}$

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5. What is the force per unit length between the two parrallel
long straight current carrying wires shown in Figure 5? ______________________________

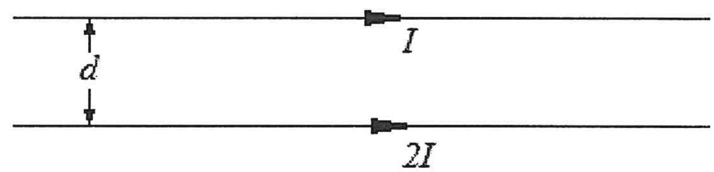

Figure 5

$$d\vec{F} = I\vec{dl} \times \vec{B}$$

$$B = \frac{\mu_0 2I}{2\pi d}$$

$$F = ILB$$

$$\frac{F}{L} = IB = \frac{\mu_0 2I^2}{2\pi d}$$

$$\boxed{\frac{F}{L} = \frac{\mu_0 I^2}{\pi d}}$$

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6. Two concentric metal spherical shells, of radius a and b, respectively, are separated by a weakly conducting material of conductivity σ as shown in Figure 6.

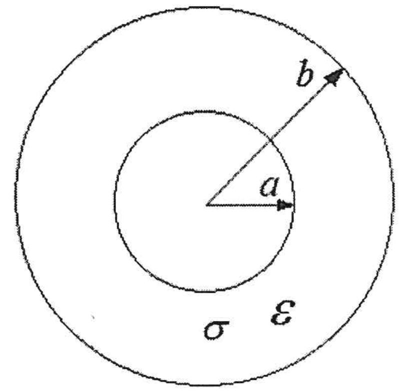

Figure 6

   a. What is the resistance between the shells? $R = \frac{b-a}{4\pi\sigma ab}$

   b. What is the capacitance between the shells? $C = 4\pi\epsilon \frac{ba}{(b-a)}$

$R = \frac{V}{I}$

$V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$

$V = \frac{q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)$

$I = \oint J \cdot da = \sigma \oint E \cdot da = \frac{\sigma}{\epsilon} Q_{enc} = \frac{\sigma q}{\epsilon}$

$R = \frac{V}{I} = \frac{\frac{q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)}{\sigma q / \epsilon_0}$

$$R = \frac{b-a}{4\pi\sigma ab}$$

$C = \frac{Q}{V} = \frac{q}{\frac{q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)} = 4\pi\epsilon\left(\frac{ab}{b-a}\right)$

Note $RC = \frac{\epsilon}{\sigma}$