PHYS 350 E&M
Exam 3

GRADES
100
100
47
92
86

February 22, 2017

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1. Find the potential inside and outside a uniformly charged solid sphere whose radius is R and whose charge is q. Use infinity as your reference point.

$$\boxed{r > R \quad V(r) = \frac{q}{4\pi\epsilon_0 r}}$$

Since the charge appears to be concentrai at the ORIGIN

$r < R$

$$\oint \vec{E} \cdot d\vec{a} = 4\pi r^2 E_r = \frac{Q_{enc}}{\epsilon_0}$$

$$V(a) - V(b) = -\int_b^a E \cdot dr$$

$$Q_{enc} = \rho \frac{4}{3}\pi r^3$$

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$Q_{enc} = q\frac{r^3}{R^3}$$

$$E = \frac{Q_{enc}}{\epsilon_0 4\pi r^2} = \frac{q \; r}{4\pi\epsilon_0 R^3}$$

$$V(r) - V(R) = \frac{-q}{4\pi\epsilon_0 R^3}\int_R^r r\,dr = \frac{-q\,(r^2 - R^2)}{8\pi\epsilon_0 R^3}$$

$$V(R) = \frac{q}{4\pi\epsilon_0 R}$$

$$V(r) = \frac{q}{4\pi\epsilon_0 R}\left[1 - \frac{1}{2}\left(\frac{r^2}{R^2} - 1\right)\right]$$

$$\boxed{V(r) = \frac{q}{8\pi\epsilon_0 R}\left(3 - \frac{r^2}{R^2}\right) \quad r < r}$$

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2. Three charges are situated at the corners of a square (side a), as shown in Figure 2. How much work does it take to bring in another charge, $+q$, from far away and place it in the forth corner?

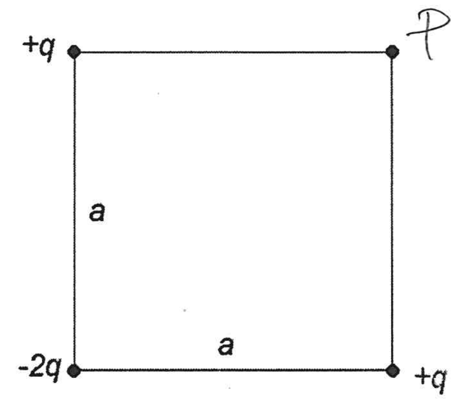

Figure 2

$$W = q V(P)$$

$$V(P) = \frac{2q}{4\pi\epsilon_0 a}\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$\boxed{W = q V(P) = \frac{q^2}{2\pi\epsilon_0 a}\left[1 - \frac{1}{\sqrt{2}}\right]}$$

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3. Find the potential $V(x,y)$ in the region $0 < y < a$ and $x > 0$, (between the two lines) shown in Figure 3.

   - $V = 0 \quad y = 0$
   - $V = 0 \quad y = a$
   - $V = 0 \quad x = \text{infinity}$
   - $V = V_o = \textit{constant} \quad x = 0$

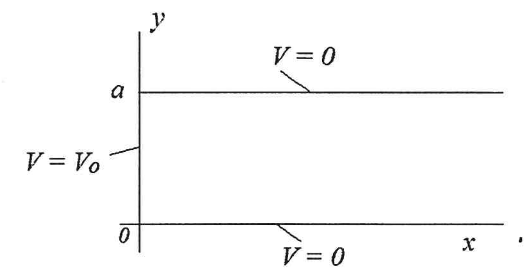

Figure 3

$$V(x,y) = X(x)\,Y(y) = \left(A_1 e^{kx} + A_2 e^{-kx}\right)\left(B_1 \cos ky + B_2 \sin ky\right)$$

Since $V = 0$ @ $x = \infty$ $A_1 = 0$

$V = 0$ @ $y = 0$ $B_1 = 0$

$V = 0$ @ $y = a$ $K = \frac{n\pi}{a}$

$$V(x,y) = c\, e^{-\frac{n\pi}{a}x} \sin\left(\frac{n\pi}{a} y\right)$$

to satisfy the boundary condition at $x = 0$

Sum solutions

$$V(0,y) = V_o = \sum C_n \sin\left(\frac{n\pi}{a} y\right)$$

$$C_n = \frac{2}{a}\int_0^a V_o \sin\frac{n\pi}{a} y\, dy = \frac{4 V_o}{a \frac{n\pi}{a}} = \frac{4V_o}{n\pi}$$

odd n

$C_n = 0$ for even

$$\boxed{V(x,y) = \frac{4V_o}{\pi} \sum_{\text{odd } n} \frac{e^{-\frac{n\pi}{a}x}}{n} \sin\frac{n\pi}{a} y}$$