GRADES

99

94

84

79

75

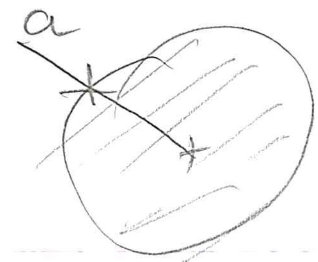
72

PHYS 350 E&M
Exam 2

February 8, 2017

Name: J. C. DALY

1. Find the electric field produce by sphere of radius *a* carring a uniform charge density ρ coul/m^3.



a. for $r < a$, $E =$ $\rho r / 3\epsilon_0$

b. for $r > a$, $E =$ $\rho a^3 / 3\epsilon_0 r^2$

$$\oint \vec{E} \cdot d\vec{a} = 4\pi r^2 E_r = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{enc} = \int \rho \, d\tau = \rho \int_0^r 4\pi r^2 dr \qquad r < a$$

$$Q_{enc} = \rho \frac{4\pi r^3}{3} \Big|_0^r = \rho \frac{4\pi r^3}{3} \qquad r < a$$

$$Q_{enc} = \frac{4\pi a^3}{3} \rho \qquad r > a$$

$$E_r = \frac{Q_{enc}}{4\pi\epsilon_0 r^2} = \frac{\rho 4\pi r^3}{3 \times 4\pi\epsilon_0 r^2} = \frac{\rho r}{3\epsilon_0} \qquad r < a$$

$$E_r = \frac{\rho 4\pi a^3}{3 \times 4\pi \epsilon_0 r^2} = \frac{\rho a^3}{3\epsilon_0 r^2} \qquad r > a$$

$$\vec{E} = E_r \hat{r}$$

$$E_r = \begin{cases} \dfrac{\rho r}{3\epsilon_0} & r < a \\ \dfrac{\rho a^3}{3\epsilon_0 r^2} & r > a \end{cases}$$

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2. Find the electric field at *P*, the center of curvature of the semi circular arc, of radius R, carrying a λ coulombs per meter as shown in Figure 2.

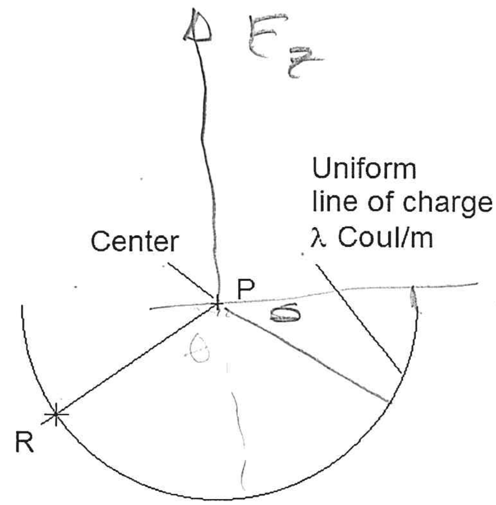

Figure 2

$$\vec{E} = \int \frac{dq\,\hat{r}}{4\pi\epsilon_0 r^2}$$

$$r = R$$

$$dq = \lambda\, dl = \lambda R\, d\theta$$

$$E_z = E \sin\theta$$

Solve for E_z since by symmetry other components equal zero

$$dE_z = \frac{\lambda R\, d\theta\, (\sin\theta)}{4\pi\epsilon_0 R^2}$$

$$= \frac{\lambda}{4\pi R\epsilon_0}\int_0^{\pi} \sin\theta\, d\theta$$

$$= \frac{\lambda}{4\pi R\epsilon_0}\Big[-\cos\theta\Big]_0^{\pi} = \frac{2\lambda}{4\pi R\epsilon_0}$$

$$\vec{E} = E_z\hat{z} = \frac{\lambda}{2\pi R\epsilon_0}\hat{z}$$

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3. Find the electric field in the region between the inner and the outer conductors of the coaxial cable shown in Figure 3. The inner conductor carries a charge of λ coul/m. The outer conductor carries a charge of $-\lambda$ coul/m. The outer radius of the inner conductor is a. The inner radius of the outer conductor is b.

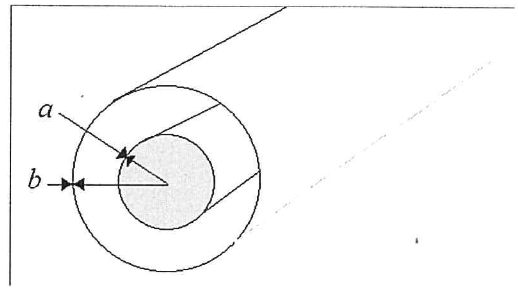
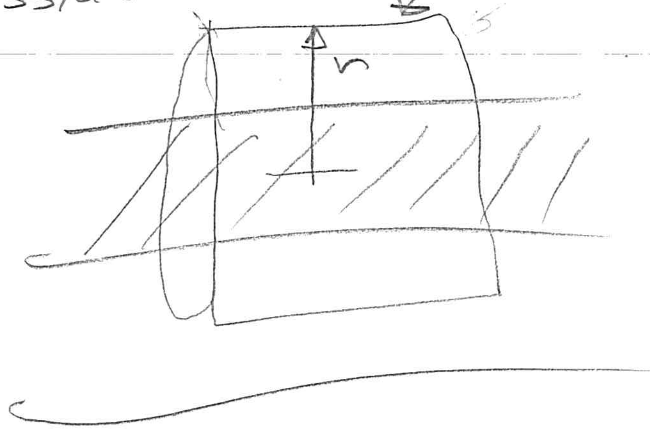


Figure 3

Find E for $a < r < b$.

Cylindrical Symmetry

$$\int \vec{E} \cdot d\hat{a} = E_r 2\pi r L$$ Gaussian Surface ?



$$Q_{enc} = \lambda L$$

$$E_r 2\pi r L = \frac{\lambda L}{\epsilon_0}$$

$$E_r = \frac{\lambda}{2\pi r \epsilon_0} \quad a < r < b$$

if $r > b$ $Q_{enc} = 0$ $\therefore E_r = 0$

$$\vec{E} = \begin{cases} \frac{\lambda}{2\pi\epsilon_0 r}\hat{r} & a < r < b \\ 0 & r > b \end{cases}$$

Between the conductors $a < r < b$

$$\boxed{\vec{E} = \frac{\lambda \hat{r}}{2\pi\epsilon_0 r}}$$