PHYS 350 E&M
Exam 1

January 25, 2017

Name J.C. Daly

1. $\vec{v} = x^2\hat{x} + 2yz\hat{y} + y^2\hat{z}$

   a. Find the divergence of ***v***.

   b. Find the curl of ***v***.

a) $\vec{\nabla}\cdot\vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 2x + 2z = 2(x+z)$

b) $\vec{\nabla}\times\vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & 2yz & y^2 \end{vmatrix}$ $\begin{matrix} \hat{x} & \hat{y} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ x^2 & 2yz \end{matrix}$

$\hat{x}(2y - 2y) + \hat{y}(0) + \hat{z}(0)$

b) $\vec{\nabla}\times\vec{v} = 0$

GRADES

100, 100

99

95

79

69

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2. $\vec{A} = 3\hat{x} + 2\hat{y}$

$\vec{B} = 3\hat{x}$

Find the angle between $\vec{A}$ and $\vec{B}$. 33.69°

$\vec{A} \cdot \vec{B} = AB\cos\theta$

$\vec{A} \cdot \vec{B} = 9 = \sqrt{13}\,3\cos\theta$

$\cos\theta = \frac{9}{3\sqrt{13}} = 0.83205$

$\theta = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right) = 33.69°$

$\overrightarrow{A \times B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 3 & 2 & 0 \\ 3 & 0 & 0 \end{vmatrix} = \hat{x}(0) + \hat{y}(0) + \hat{z}(-6)$

$= -6\hat{z}$

$6 = AB\sin\theta \qquad A = \sqrt{13} \qquad B = 3$

$6 = 3\sqrt{13}\sin\theta$

$\sin\theta = \frac{-6}{3\sqrt{13}} = \frac{-2}{\sqrt{13}}$

$\theta = \sin^{-1}\frac{-2}{\sqrt{13}} = -33.69°$

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3. The height of a certain hill (in meters) is given by

$$h(x,y) = 4xy - 3x^2 - 2y^2 - 18x + 12y + 28$$

where *y* is the distance (in miles) north, *x* is the distance east of North Hadley.

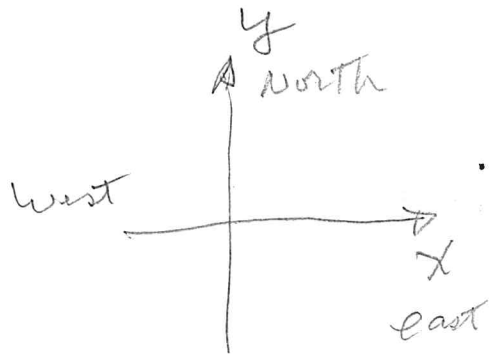
a. Where is the top of the hill located? 3 miles due west of North Hadle

b. How high is the hill? 55 meters

c. How steep is the hill (in feet per mile) at a point 2 miles north and one mile east of South Hadley? 17.9 m/mi

In what direction is the slope steepest at that location? 26.6° North of west

$$\vec{\nabla} h = \hat{x}\frac{\partial h}{\partial x} + \hat{y}\frac{\partial h}{\partial y}$$



$$\frac{\partial h}{\partial x} = 4y - 6x - 18$$

$$\frac{\partial h}{\partial y} = 4x - 4y + 12$$

a) $\vec{\nabla} h = 0$ at the Top

$$-6x + 4y - 18 = 0 = \frac{\partial h}{\partial x}$$

$$4x - 4y + 12 = 0 = \frac{\partial h}{\partial y}$$

$$-2x - 6 = 0$$

$$x = -3$$

$$-6(-3) + 4y - 18 = 0$$

$$y = 0$$

a) top of the hill is at $x = -3$, $y = 0$

3 miles west of North Hadley

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b. $h = 4xy - 3x^2 - 2y^2 - 18x + 12y + 25$

at the top $x = -3$ $y = 0$

$h = -3*3^2 - 18*(-3) + 25$ (−27, +54)

$54 + 25 = 82$, $82 - 27 = 55$

$h = 55$ meters ✓

c) Find ∇h at $x = 1$ $y = 2$

$\nabla h = (4y - 4x - 18)\hat{x} + (4x - 4y + 12)\hat{y}$

@ $x = 1$, $y = 2$

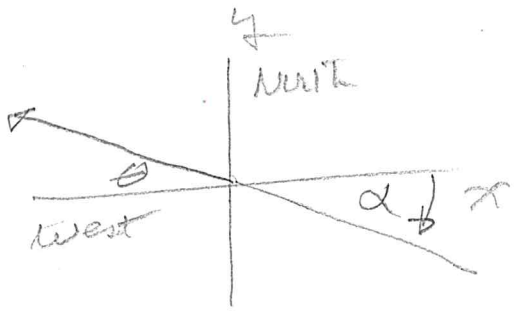
$\nabla h = (8 - 4 - 18)\hat{x} + (4 - 8 + 12)\hat{y}$

$\nabla h = -16\hat{x} + 8\hat{y}$

$|\nabla h| = 8\sqrt{4 + 1} = 8\sqrt{5} = 17.9$ ft/mi

$\alpha = \tan^{-1}\left(-\frac{1}{2}\right)$

$= -26.565°$



The direction of the steepest slope is 26.6° North of West

$\tan^{-1}\left(\frac{1}{2}\right) = 15\frac{1}{3}°$ and $-26.6°$ ✓